Photonics – 2020

First Project: The Twin Paradox

Two twins, **Alice** and **Bob**, are making an experiment that shows a very important aspect of special relativity, namely – the twin (or clock) "paradox."

Basically, the twin (or clock) paradox asserts that if one clock remains at rest in an inertial frame, and another, initially agreeing with it, is taken off on any sort of path and finally brought back to its starting point, the second clock will have lost time as compared with the first.

«The "paradox" consists in a one-sidedness that appears to flout the basic tenets of relativity. Both the traveler and the stay-at-home agree that the traveler has aged less than the other."

Anthony Philip French (1920 – 2017), *Special Relativity (The MIT Introductory Physics Series)*. New York, NY: W. W. Norton & Company, 1966 (page 155).

In this project we are assuming that Bob is the **stay-at-home** twin whereas Alice is his twin sister acting as the **astronaut**.

We are always using, in this work, natural (or geometric) units where the speed of light (in vacuum) is dimensionless and has the numerical value

$$\boxed{c = 1}\,.$$

In fact, time is measured in **years** and space in **light-years**.

When the experiment begins both twins (Alice and Bob) are $\mathscr{A} = 30$ years old. The main goal, in this project, is to clearly show that time evolves differently for the two twins. Indeed, according to Bob, the time of total trip was T. However, according to Alice, the time of total trip was T'. Therefore, both twins agree – at the end of this experiment – that Bob is $\mathscr{B} = 30 + T$ years old whereas Alice is actually $\mathscr{C} = 30 + T'$ years old, with

$$T' < T \quad \Rightarrow \quad \mathscr{C} < \mathscr{B}.$$

For numerical illustration of your calculations you should consider that

$$\boxed{\beta = \frac{4}{5}}.$$

Therefore, according to Bob, the turnaround event occurs at a distance

$$\boxed{L = \beta \frac{T}{2}}.$$

Let us consider that

$$\boxed{L = 4 \text{ light-years}}.$$

The experiment

1. The traveler (Alice) starts off, reaching a constant velocity β within a negligibly short time.
2. After journeying for a while, the traveler (Alice) suddenly reverses velocity (the turnaround event).
3. The traveler arrives back at the starting point (on Earth), and then stops.

In order to give a full explanation of the whole experiment you should present spacetime diagrams (in two-dimensional Minkowski spacetime) corresponding to each viewpoint:

- Bob's viewpoint, from $\mathscr{A} = 30$ to $\mathscr{B} = 30 + T$;
- Alice's viewpoint, from $\mathscr{A} = 30$ to $\mathscr{C} = 30 + T'$.

But it is not possible to regard Bob as the traveler and Alice as the stay-at-home? No! Why not? Because there isn't symmetry between the two. The turnaround event is the decisive one. During it, Alice switches from one inertial frame to another, while nothing at all happens to Bob. At this turnaround event, Alice experiences an acceleration which she clearly feels.

You should use the concepts of «equiloc» and «equitemp» to explain how the reciprocity of time dilation is broken at the turnaround event.

«It used to be frequently argued that it would be necessary to pass to Einstein's *general* relativity in order to get acceleration, but this is completely wrong. (…) The astronaut is allowed to accelerate in special relativity, just as in general relativity. »

Roger Penrose, *The Road to Reality – A Complete Guide to the Laws of the Universe*. New York, NY: Alfred A. Knopf, 2005 (page 422).

To analyze the whole process, you should use the **Doppler effect**. We imagine that each person sends equally spaced time signals (of his/her own proper time) to the other. The cumulative counts of time signals for the whole trip are then compared. Suppose that each person is transmitting f pulses per unit time (year). As Alice travels away from Bob, each observer will receive the other's signals at the reduced rate

$$\boxed{f' = f\sqrt{\frac{1-\beta}{1+\beta}} < f}\,.$$

But for how long? Here is the **asymmetry.** As soon as Alice reverses, she begins to receive signals from Bob at the enhanced rate

$$\boxed{f'' = f\sqrt{\frac{1+\beta}{1-\beta}} > f}\,.$$

With Bob it is quite different. The last signal sent by Alice before she reverses does not reach Bob until a time $t_1 = L$ (according to his proper time) later. Thus for much more than one-half the total time, Bob is recording Alice's signals at a lower rate f'. Only in the latter stages does Bob receive pulses at the higher rate f''. We can show that that each observer receives as many signals as the other sends between star and finish of the trip.

They agree to disagree about their respective measures of the total time.